

Coulomb corrections in the extraction of the proton radius

John Arrington¹

¹*Physics Division, Argonne National Laboratory, Argonne, IL, 60439, USA*

Multi-photon exchange contributions are important in extracting the proton charge radius from elastic electron-proton scattering. So far, only diagrams associated with the exchange of a second photon have been evaluated. At the very low Q^2 values relevant to the radius extraction, higher order contributions may become important. We evaluate these corrections in the Effective Momentum Approximation, which includes the Coulomb interaction to all orders, and find small corrections with a strong Q^2 dependence at low Q^2 and large scattering angles. This suggests that the higher order terms may be important in the evaluation of the proton magnetic radius.

PACS numbers: 13.40.Gp, 14.20.Dh, 25.30.Bf

The proton RMS charge radius, R_p , has become a topic of great interest following the recent Lamb shift measurement in muonic hydrogen [1] which is extremely sensitive to R_p . This measurement yields $R_p=0.8418(7)$ fm, significantly smaller than recent extractions based on electron-proton interactions [2–5]. These include both electron scattering and electron-proton interactions in the hydrogen atom, which yield a combined result of $R_p=0.8772(46)$ [5, 6]. Replacing the CODATA06 value from Ref. [3] with the updated CODATA10 value [7] yields an electron-based average of $0.8775(39)$, corresponding to a 9σ difference between the muonic hydrogen extraction and the combined electron measurements.

Radiative corrections play an important role in the electron scattering measurements. While the largest corrections are well understood, the diagrams which depend on the proton structure, e.g. the two-photon exchange (TPE) diagrams, cannot be calculated exactly due to the hadronic structure uncertainty. In the past, these corrections were calculated in the 2nd Born approximation, assuming the exchange of a second soft photon with an unexcited intermediate state. Initial calculations were performed in the limit $Q^2 \rightarrow 0$ [8], and later at finite Q^2 , which require a parameterization of the proton charge and magnetic form factors, G_E and G_M . The inclusion of these corrections was found to be important in the extraction of the proton radius [2, 9].

More recently, calculations going beyond the 2nd Born approximation have been performed [10–16], motivated by the discrepancy between Rosenbluth and polarization measurements at high Q^2 [17–21]. See recent reviews [22, 23] for further details on the different theoretical approaches. There have also been several phenomenological extractions of TPE contributions [24–30] and attempts to experimentally constrain TPE [31–37], but essentially all of these works focus on $Q^2 > 2\text{--}3 \text{ GeV}^2$, where there is a clear discrepancy between the Rosenbluth and polarization extractions of the form factors.

Focusing on the low Q^2 regime, Figure 1 shows the range of TPE corrections to the cross section based on a set of calculations which include both hard and soft TPE and which aim to be reliable at low Q^2 : The hadronic

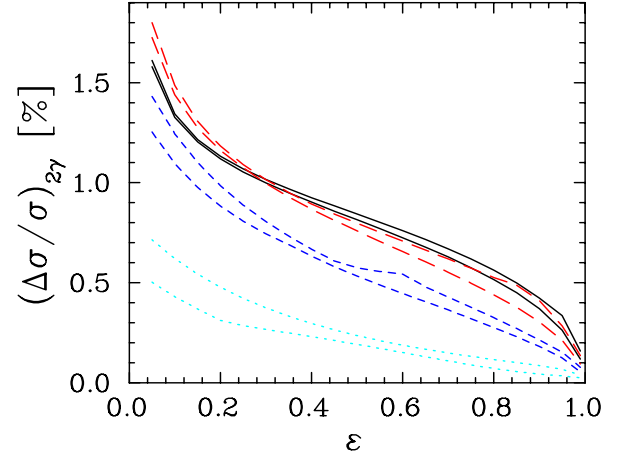


FIG. 1: (color online) Range of results for several TPE calculations at low Q^2 [11, 13, 14, 38, 39]. Solid, long-dashed, short-dashed, and dotted lines show the range for $Q^2=0.01, 0.03, 0.1$, and 0.2 GeV^2 , respectively. Note that the low Q^2 expansion [38] is only valid up to $Q^2=0.1 \text{ GeV}^2$, and so is excluded from the $Q^2 = 0.2 \text{ GeV}^2$ range.

calculation of Blunden, Melnitchouk, and Tjon [11], a similar calculation by Borisjuk and Kobushkin [13], their low- Q^2 TPE expansion [38], and their dispersion calculation excluding [14] and including [39] Δ resonance contributions. While the correction decreases with increasing Q^2 for $\varepsilon > 0.4$, the TPE correction for $\varepsilon < 0.3$ first increases and then decreases. These corrections have been shown to have an important effect on the form factor extraction at low Q^2 values [40], as well as impacting the extracted charge and magnetization radii of the proton [5, 13, 41–43]. While the calculations use different approaches and have different input, they are in good agreement with a typical spread between calculations below 0.05% , suggesting that the TPE corrections are well known in this region. At these low Q^2 values, deviations from a linear correction in ε are small except for extreme ε values, making tests of the linearity of the Rosenbluth separation [26, 32] fairly insensitive to the low Q^2 contributions of these calculations.

Extraction of the radius requires cross sections measured at low Q^2 , meaning low electron energies, especially in the case of large-angle measurements needed to constrain G_M and the magnetization radius. At low energies, the change in the electron energy due to its interaction with the Coulomb potential of the nucleus may yield an important change in the kinematics at the scattering vertex. In quasi-elastic scattering this effect is sometimes estimated in the Effective Momentum Approximation (EMA) [44], where the acceleration due to the Coulomb field of a high- Z nucleus can have a significant impact on the e-p scattering kinematics. This approach has the advantage of including the Coulomb interaction to all orders in a semi-classical picture, and can easily be applied to both high- and low-energy kinematics. This is to be compared to the 2nd Born approximation which is a more rigorous approach, but which includes only two-photon exchange.

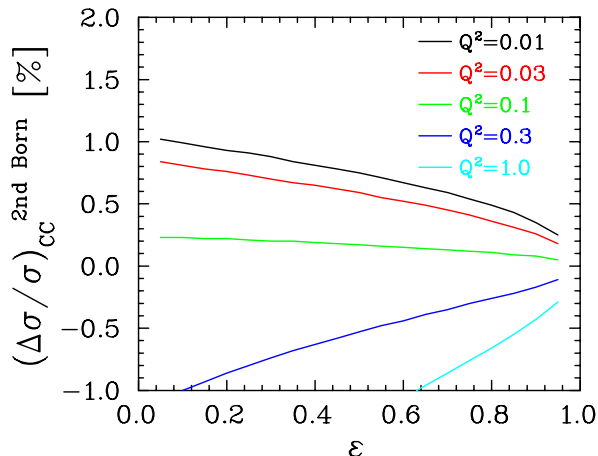


FIG. 2: (color online) Coulomb corrections calculated in the 2nd Born approximation for e-p elastic scattering for Q^2 values from 0.01 GeV^2 (top curve) to 1 GeV^2 (bottom curve).

Figure 2 shows the fractional correction to the cross section in the 2nd Born approximation. One can make a crude estimate of the impact of these corrections on the radius. The RMS radius, R_p , is defined in terms of the low Q^2 expansion of the form factor, $G_E(Q^2) \approx 1 - Q^2 R_p^2/6$, with a similar expression for the magnetic radius. A proton RMS radius of 0.85 fm yields $G_E(Q^2) \approx 1 - 3Q^2$, with Q^2 in GeV^2 , or a fractional slope at $Q^2 = 0$ of roughly 300%/GeV². The charge form factor contribution to the cross section goes like $G_E^2(Q^2)$, for a slope of roughly 600%/GeV² in the cross section. Coulomb corrections in the 2nd Born approximation change the cross section by about 0.2% between $Q^2=0.01$ and 0.03 GeV^2 (Fig. 2), yielding a change in the slope of 0.2%/0.02 $\text{GeV}^2=10\%/\text{GeV}^2$. This is roughly 2% of the total slope introduced by the protons size, corresponding to a 1% change in the extracted radius, in good agreement with the observation of a roughly 0.01 fm

change observed when the 2nd Born calculation is applied to the data [9].

In this paper, we estimate the impact of higher-order Coulomb corrections by applying the EMA prescription of Ref. [45] to elastic e-p scattering at low Q^2 . The key parameter in the calculation is the Coulomb potential at the point where the scattering occurs. When the EMA is used to evaluate scattering from a heavy nucleus, it is generally assumed that the scattering occurs uniformly within the nucleus, so the Coulomb potential is taken to be something between the surface and central potential. Averaging over the nuclear volume, assuming a uniform charge sphere, yields a potential that is 80% of the central potential, in good agreement with the result of Ref. [44] which adjusts the average potential in the EMA calculations to match distorted-wave Born approximation results [46, 47] which are more reliable but can only be performed at lower Q^2 values.

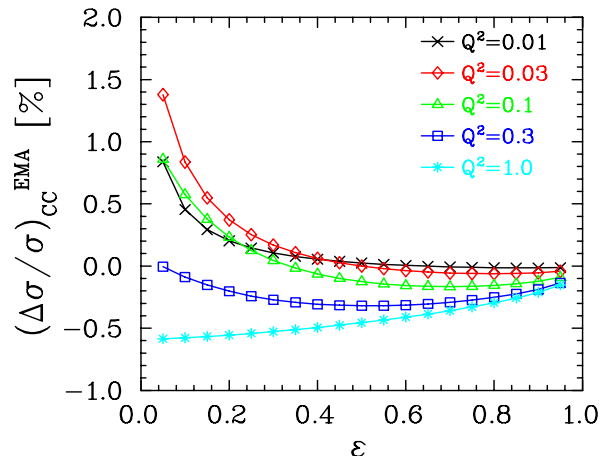


FIG. 3: (color online) Coulomb corrections calculated in the Effective Momentum Approximation (as described in the text) for e-p elastic scattering for $0.01 < Q^2 < 1 \text{ GeV}^2$.

At high energies, the EMA calculation yields a correction that is very similar to the 2nd Born approximation for $\varepsilon > 0.5$, as shown in Ref. [45], with differences the 1–2% level for larger-angle scattering. Note that the EMA calculation applied in Ref. [45] used the central potential and did not include the flux factor of Ref. [44]. Inclusion of this correction yields somewhat improved agreement at low ε , but there are still clear differences remaining.

For e-p elastic scattering at very low Q^2 , one expects that the scattering will occur when the electron is outside of the proton, so using the average Coulomb potential in the proton would be an overestimate. We take the Coulomb potential corresponding to the case where the scattering occurs at a separation $\Delta x = 1/q$, where q is the momentum of the exchanged virtual photon. This will suppress the impact of the energy shift for Q^2 values where the scattering occurs outside of the proton. This

occurs for $Q^2 < 0.03 \text{ GeV}^2$ if we take the proton to be a uniform sphere of radius 1.15 fm to match the observed RMS radius. For Q^2 values where the scattering occurs inside of the proton, we limit the Coulomb potential to a maximum of the volume-averaged value of 1.5 MeV.

Figure 3 shows the Coulomb correction in the EMA. The behavior is qualitatively similar to the 2nd Born approximation result for $Q^2 = 1 \text{ GeV}^2$, as it is for larger Q^2 values [45]. The EMA result has a very different angular dependence at very low Q^2 values, with a sharp rise at low ε . As such, this correction could have an important impact on the low Q^2 extraction of the form factors and radius, especially for the extraction of G_M and the magnetic radius.

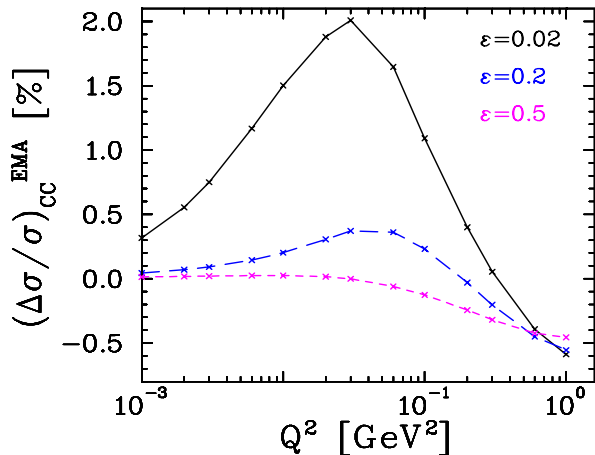


FIG. 4: (color online) Q^2 dependence of the EMA correction at $\varepsilon=0.02$ (solid), 0.2 (long dash), and 0.5 (short dash).

Figure 4 shows the EMA result as a function of Q^2 for three ε values. At larger ε values, the effect is small, especially near $Q^2=0$. For $\varepsilon = 0.02$, corresponding to scattering near 180 degrees, there is a very rapid rise with Q^2 (note that Q^2 is shown on a logarithmic scale). From the Q^2 dependence of the correction, it is straightforward to estimate what impact this would have on a direct extraction of the magnetic radius from very low Q^2 data at $\varepsilon \approx 0$. The EMA yields a 1% change in the correction for a change in Q^2 of less than 0.01 GeV^2 , or a change in the slope of $100\%/Q^2$. This would correspond to a very large correction to the magnetic radius.

However, existing measurements below $Q^2=0.1 \text{ GeV}^2$ are limited and there are not measurements over this Q^2 and ε range. The kinematics of interest are at somewhat larger Q^2 or ε values. At $\varepsilon=0.02$, the correction decreases by 1.5% going from $Q^2 = 0.03 \text{ GeV}^2$ to 0.2 GeV^2 , while for $\varepsilon = 0.2$, it increases 0.3% in going from 0.01 to 0.03 GeV^2 . These provide changes in the fractional slope of the cross section of $-8\%/ \text{GeV}^2$ and $+15\%/ \text{GeV}^2$, respectively, corresponding to small but potentially significant corrections to the extracted magnetic proton radius. The corrections at larger Q^2 and ε values are smaller, but

these kinematics require extrapolations to $\varepsilon = 0$ and low Q^2 , and thus even these smaller corrections may not be entirely negligible.

While this rough estimate shows that these effects could have important contributions to the extraction of the magnetic radius, it is difficult to provide a more detailed estimate of the impact. First, because it is very sensitive to the exact kinematics of the data included in the extraction. The corrections would have to be applied to each measurement and then the radius extraction procedure, including fitting of normalization factors if they are allowed to vary, would have to be repeated. In addition, it is not clear how one would combine the results of the EMA at very low Q^2 with the TPE calculations, to include the full impact of the corrections without double counting. A more detailed calculation, e.g. in the distorted-wave Born approximation, would allow for a more reliable evaluation of the importance of the higher order contributions.

In conclusion, we find a clear difference between the 2nd Born approximation and the EMA evaluation of Coulomb corrections for e-p elastic scattering at low Q^2 . The difference is particularly important large angles, where the data are sensitive to $G_M(Q^2)$, suggesting that effects beyond two-photon exchange may be important in the extraction of the magnetic radius. A realistic estimate of their impact would involve a more detailed calculation, with the impact of the correction evaluated using on the exact data set and fitting procedure used to extract the radius.

I thank M. K. Medina checking the calculations presented here, and P. Blunden and B. Kobushkin for providing calculations and useful discussion. This work was supported by the U.S. DOE through contract DE-AC02-06CH11357.

-
- [1] R. Pohl et al., Nature **466**, 213 (2010).
 - [2] I. Sick, Phys. Lett. B **576**, 62 (2003).
 - [3] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008).
 - [4] J. C. Bernauer et al., Phys. Rev. Lett. **105**, 242001 (2010).
 - [5] X. Zhan et al., Phys.Lett. **B705**, 59 (2011).
 - [6] G. Ron et al., Phys. Rev. C **84**, 055204 (2011).
 - [7] P. J. Mohr, B. N. Taylor, and D. B. Newell (2012), arXiv:1203.5425.
 - [8] W. A. McKinley and H. Feshbach, Phys. Rev. **74**, 1759 (1948).
 - [9] R. Rosenfelder, Phys. Lett. B **479**, 381 (2000).
 - [10] L. C. Maximon and J. A. Tjon, Phys. Rev. C **62**, 054320 (2000).
 - [11] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, Phys. Rev. C **72**, 034612 (2005).
 - [12] A. V. Afanasev, S. J. Brodsky, C. E. Carlson, Y.-C. Chen, and M. Vanderhaeghen, Phys. Rev. D **72**, 013008 (2005).

- [13] D. Borisyuk and A. Kobushkin, Phys. Rev. C **74**, 065203 (2006).
- [14] D. Borisyuk and A. Kobushkin, Phys. Rev. C **78**, 025208 (2008).
- [15] N. Kivel and M. Vanderhaeghen, Phys. Rev. Lett. **103**, 092004 (2009).
- [16] D. Borisyuk and A. Kobushkin, Phys. Rev. D **79**, 034001 (2009).
- [17] J. Arrington, Phys. Rev. C **68**, 034325 (2003).
- [18] P. A. M. Guichon and M. Vanderhaeghen, Phys. Rev. Lett. **91**, 142303 (2003).
- [19] J. Arrington, C. D. Roberts, and J. M. Zanotti, J. Phys. **G34**, 23 (2007).
- [20] C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen, Prog. Part. Nucl. Phys. **59**, 694 (2007).
- [21] J. Arrington, K. de Jager, and C. F. Perdrisat, J. Phys. Conf. Ser. **299**, 012002 (2011), arXiv:1102.2463.
- [22] C. E. Carlson and M. Vanderhaeghen, Ann. Rev. Nucl. Part. Sci. **57**, 171 (2007).
- [23] J. Arrington, P. Blunden, and W. Melnitchouk, Prog. Part. Nucl. Phys. **66**, 782 (2011).
- [24] J. Arrington, Phys. Rev. C **71**, 015202 (2005).
- [25] D. Borisyuk and A. Kobushkin, Phys. Rev. C **76**, 022201 (2007).
- [26] Y.-C. Chen, C.-W. Kao, and S.-N. Yang, Phys. Lett. **B652**, 269 (2007).
- [27] M. Belushkin, H.-W. Hammer, and U.-G. Meissner, Phys. Lett. **B658**, 138 (2008).
- [28] K. M. Graczyk, Phys. Rev. C **84**, 034314 (2011).
- [29] I. A. Qattan and A. Alsaad, Phys. Rev. C **83**, 054307 (2011).
- [30] I. A. Qattan, A. Alsaad, and J. Arrington, Phys. Rev. C **84**, 054317 (2011).
- [31] I. A. Qattan et al., Phys. Rev. Lett. **94**, 142301 (2005).
- [32] V. Tvaskis et al., Phys. Rev. C **73**, 025206 (2006).
- [33] J. Arrington, D. M. Nikolenko, et al., Proposal for positron measurement at VEPP-3, nucl-ex/0408020.
- [34] J. Arrington, AIP Conf. Proc. **1160**, 13 (2009), arXiv:0905.0713.
- [35] D. Borisyuk and A. Kobushkin, Phys. Rev. D **83**, 057501 (2011).
- [36] J. Guttman et al., arXiv:1012.0564 (2011).
- [37] M. Meiziane et al., Phys. Rev. Lett. **106**, 132501 (2011).
- [38] D. Borisyuk and A. Kobushkin, Phys. Rev. C **75**, 038202 (2007).
- [39] D. Borisyuk and A. Kobushkin (2012), arXiv:1206.0155.
- [40] J. Arrington, W. Melnitchouk, and J. A. Tjon, Phys. Rev. **C76**, 035205 (2007).
- [41] P. G. Blunden and I. Sick, Phys. Rev. C **72**, 057601 (2005).
- [42] J. Arrington, Phys. Rev. Lett. **107**, 119101 (2011).
- [43] J. Bernauer et al., Phys. Rev. Lett. **107**, 119102 (2011).
- [44] A. Aste, C. von Arx, and D. Trautmann, Eur. Phys. J. **A26**, 167 (2005).
- [45] J. Arrington and I. Sick, Phys. Rev. C **70**, 028203 (2004).
- [46] K. Kim, L. Wright, Y. Jin, and D. Kosik, Phys. Rev. C **54**, 2515 (1996).
- [47] J. Udias, P. Sarriuren, E. Moya de Guerra, E. Garrido, and J. Caballero, Phys. Rev. C **48**, 2731 (1993).